CHAPTER 9

The Capital Asset Pricing Model
Capital Asset Pricing Model (CAPM)

• It is the *equilibrium* model that underlies all modern financial theory
• Derived using principles of diversification with simplified assumptions
• Markowitz, Sharpe, Lintner and Mossin are researchers credited with its development
Individual investors are price takers

Single-period investment horizon

Investments are limited to traded financial assets

No taxes and no transaction costs

Borrow at $r_f$

Information is costless and available to all investors

Investors are rational mean-variance optimizers

Expectations are homogeneous
Resulting Equilibrium Conditions

- The market portfolio (M) is on the efficient frontier and is on the Capital Market Line.
- All investors will hold the same portfolio for risky assets – market portfolio (M).
- Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value (total market value = total wealth).
Resulting Equilibrium Conditions

• Risk premium on the market depends on the average risk aversion of all market participants

• Risk premium on an individual security is a function of its covariance with the market
Figure 9.1 The Efficient Frontier and the Capital Market Line
Market Risk Premium

The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the investor:

\[ E(r_M) - r_f = \bar{A} \sigma_M^2 \]

Where

- \( \sigma_M^2 \) = variance of the Market portfolio
- \( \bar{A} \) = average degree or risk aversion

Q. Where does this formula come from?
Return and Risk For Individual Securities

• The risk premium on individual securities is a function of the individual security’s contribution to the risk of the market portfolio.

• An individual security’s risk premium is a function of the covariance of returns with the assets that make up the market portfolio.

• Why bother doing security analysis?
GE Example

• Covariance of GE return with the market portfolio:

\[
\sum_{i=1}^{n} w_i \text{Cov}(R_i, R_{GE}) = \sum_{i=1}^{n} \text{Cov}(w_i R_i, R_{GE})
\]

\[
= \text{Cov} \left( \sum_{i=1}^{n} w_i R_i, R_{GE} \right)
\]

• Therefore, the reward-to-risk ratio for investments in GE would be:

\[
= \frac{\text{GE's contrib.to risk premium}}{\text{GE's contrib.to variance}} = \frac{w_{GE} E[R_{GE}]}{w_{GE} \text{Cov}(R_{GE}, R_M)} = \frac{E[r_{GE}] - r_f}{\text{Cov}(R_{GE}, R_M)}
\]
GE Example

- Reward-to-risk ratio for investment in market portfolio:

\[
\frac{\text{Market risk premium}}{\text{Market variance}} = \frac{E(r_M) - r_f}{\sigma^2_M}
\]

- At equilibrium all reward-to-risk ratios are equal, including that of GE:

\[
\frac{E(r_{GE}) - r_f}{\text{Cov}(r_{GE}, r_M)} = \frac{E(r_M) - r_f}{\sigma^2_M}
\]
GE Example

• The risk premium for GE:

\[ E[r_{GE}] - r_f = \frac{COV(R_{GE}, R_M)}{\sigma^2_M} \left[ E[r_M] - r_f \right] \]

• Restating, we obtain:

\[ E[r_{GE}] = r_f + \beta_{GE} \left( E[r_M] - r_f \right) \]
Expected Return-Beta Relationship

- CAPM holds for the overall portfolio because:

\[ E[r_P] = \sum_k w_k E[r_k] \]

and

\[ \beta_P = \sum_k w_k \beta_k \]

- This also holds for the market portfolio:

\[ E[r_M] = r_f + \beta_M (E[r_M] - r_f) \]

(remember $\beta_M = 1$)
Fig. 9.2 The Security Market Line (SML)
Figure 9.3 The SML and a Positive-Alpha Stock
The Index Model and Realized Returns

• CAPM is based on *expected* returns:

\[ E[r_i] = r_f + \beta_i \left( E[r_M] - r_f \right) \]

• To move from expected to *realized* returns, use the index model in excess return form:

\[ R_i = \alpha_i + \beta_i R_M + e_i \]

• Observe that index model beta is the same as the beta of the CAPM (quick derivation)

• Compare the two: should \( \alpha \) be zero?
Figure 9.4 Estimates of Individual Mutual Fund Alphas, 1972-1991

- CAPM: $E[\alpha_i] = 0 \ \forall i$
- index: *realized* $\alpha$ should average to zero

Mean $< 0$ (slightly)
but statistically indistinguishable from zero
Is the CAPM Practical?

• CAPM is a good model to explain *expected* returns on risky assets. This means:
  – Without security analysis, $\alpha$ is assumed to be zero
  – Positive and negative alphas are revealed only by superior security analysis
Is the CAPM Practical?

- CAPM assumes the market portfolio M is mean-variance optimal.
- Must use a proxy for market portfolio (for example, but not limited to, S&P500)
- CAPM is still considered the best available description of security pricing and is widely accepted (i.e. assume $\alpha=0$ w/out analysis)
Is the CAPM Testable?

- Empirical tests reject hypothesis $\alpha = 0$
- Low $\beta$ securities have $\alpha > 0$
- High $\beta$ securities have $\alpha < 0$
- Is CAPM then not valid?
- No better model out there, we measure $\alpha$ and $\beta$ with unsatisfactory precision
- No mutual fund \textit{consistently} outperforms the passive strategy
Econometrics and the Expected Return-Beta Relationship

• Are empirical tests poorly designed?
• Statistical bias is easily introduced
• Miller and Scholes paper demonstrated how econometric problems could lead one to reject the CAPM even if it were perfectly valid
• For example residuals are correlated within the same industry
Extensions of the CAPM

• Zero-Beta Model
  – Combine frontier-portfolios to obtain portfolios also on the efficient frontier
  – Uncorrelated pairs of top and bottom efficient frontier portfolio
  – Helps explain $\alpha > 0$ for low $\beta$ stocks and $\alpha < 0$ on high $\beta$ stocks

• Consideration of labor income and non-traded assets (e.g. private equity)
Extensions of the CAPM

- Consumption-based CAPM (Rubinstein, Lucas, Breeden)
- Investors allocate wealth between consumption today and investment for the future (future wealth comes from investment and labor)

- Merton’s Multiperiod Model and hedge portfolios (ICAPM)
- Incorporation of the effects of changes in the real rate of interest and inflation
- K factors generalize SML to Multi-index model
Liquidity and the CAPM

- **Liquidity**: The ease and speed with which an asset can be sold at fair market value.
- **Illiquidity Premium**: Discount from fair market value the seller must accept to obtain a (quick) sale.
  - Measured partly by bid-asked spread.
  - As trading costs are higher, the illiquidity discount will be greater.
Figure 9.5 The Relationship Between Illiquidity and Average Returns
Liquidity Risk

- In a financial crisis, liquidity can unexpectedly dry up.
- When liquidity in one stock decreases, it tends to decrease in other stocks at the same time.
- Investors demand compensation for liquidity risk
  - Liquidity betas
CAPM and the Real World

• Academic world
  – Cannot observe all tradable assets
  – Impossible to pin down market portfolio
  – Attempts to validate using regression analysis

• Investment Industry
  – Relies on the single-index CAPM model
  – Most investors don’t beat the index portfolio