CHAPTER 10

Arbitrage Pricing Theory and Multifactor Models of Risk and Return
Single Factor Model

• Returns on a security come from two sources:
  – Common macro-economic factor
  – Firm specific events

• Possible common macro-economic factors
  – Gross Domestic Product Growth
  – Interest Rates
  – What else?
Single Factor Model Equation

\[ R_i = \text{Excess Return on security} \]
\[ R_i = E[R_i] + \beta_i F + e_i \]

\( \beta_i = \text{Factor sensitivity, or factor loading, or factor beta} \)

\( F = \text{Surprise in macro-economic factor} \)
\( F \) could be positive or negative but has expected value of zero.

\( e_i = \text{Firm specific events (zero expected value)} \)
\( (\text{uncorrelated with each other and with } F) \)
Multifactor Models

• Use more than one factor in addition to market return. Examples include:
  – gross domestic product
  – expected inflation
  – interest rates
  – …

• Estimate a beta or factor loading for each factor using multiple regression.
Multifactor Model Equation

\[ R_i = \text{Actual excess return for security } i \]

\[ R_i = E[R_i] + \beta_{i,GDP} GDP + \beta_{i,IR} IR \]

\[ \beta_{i,GDP} = \text{Factor sensitivity for GDP} \]

\[ \beta_{i,IR} = \text{Factor sensitivity for Interest Rate} \]

\[ e_i = \text{Firm specific events} \]
Multifactor SML Models

\[ E[r_i] = r_f + \beta_{i,GDP}RP_{GDP} + \beta_{i,IR}RP_{IR} \]

- \( \beta_{i,GDP} \) = Factor sensitivity for GDP
- \( RP_{i,GDP} \) = Risk premium for GDP
- \( \beta_{i,IR} \) = Factor sensitivity for Interest Rate
- \( RP_{i,IR} \) = Risk premium for Interest Rate
Interpretation

The expected return on a security is the sum of:

1. The risk-free rate
2. The sensitivity to GDP times the risk premium for bearing GDP risk
3. The sensitivity to interest rate risk times the risk premium for bearing interest rate risk
Arbitrage Pricing Theory

1. Securities described with a Factor Model
2. There are enough securities to diversify away idiosyncratic risk
3. Arbitrage will disappear quickly
   • Arbitrage when a zero investment portfolio has a sure profit
   • No investment is required so investors can create large positions to obtain large profits
Arbitrage Pricing Theory

- Regardless of wealth or risk aversion, investors will want an infinitely large position in the risk-free arbitrage portfolio.

- In efficient markets, profitable arbitrage opportunities will quickly disappear.
APT & Well-Diversified Portfolios

\[ R_P = E[R_P] + \beta_P F + e_P \]

\( F = \text{some factor} \)

- For a well-diversified portfolio, \( e_P \):
  - approaches zero as the number of securities in the portfolio increases
  - and their associated weights decrease
Figure 10.1 Returns as a Function of the Systematic Factor

Well-diversified portfolio and single stock
Can the two (well diversified) portfolios co-exist?

Figure 10.2 Returns as a Function of the Systematic Factor: An Arbitrage Opportunity
Figure 10.3 An Arbitrage Opportunity
Figure 10.4 The Security Market Line

- Expected Return (%)
- $E(r_M)$
- $r_f$
- $M$
- $[E(r_M) - r_f]$
- $\beta$ (With respect to market index)
APT Model

\[ E[R_P] = \beta_P E[R_M] \]

- APT applies to well diversified portfolios and not necessarily to individual stocks.
- It implies \( \alpha = 0 \)
- With APT it is possible for some individual stocks to be mispriced – not lie on the SML, although APT must hold for most stocks (proof is difficult but reasoning can be illustrated)
- APT can be extended to multifactor models.
APT and CAPM

APT
• Equilibrium means no arbitrage opportunities.
• APT equilibrium is quickly restored upon arbitrage.
• Assumes a diversified portfolio, but residual risk is still a factor.
• Does not assume investors are mean-variance optimizers.
• Reveals arbitrage opportunities.

CAPM
• Model is based on an inherently unobservable “market” portfolio.
• Rests on mean-variance efficiency. The actions of many small investors restore CAPM equilibrium.
• CAPM describes equilibrium for all assets.
Multifactor APT

• Use of more than a single systematic factor
• Requires formation of factor portfolios
• What factors to choose?
  – Factors that are important to performance of the general economy
  – What about firm characteristics?

  – (example)
Two-Factor Model

\[ R_i = E[R_i] + \beta_{i,1} R_i + \beta_{i,2} F_2 + e_i \]

• The multifactor APT is similar to the one-factor case.
• Each factor \( F \) has zero expected value as it measures the surprise, not the level.
• Also \( e_i \) has zero expected value.
Two (or multi)-Factor Model

- Track with diversified *factor portfolios*
- The factor portfolios track a particular source of macroeconomic risk, but are uncorrelated with other sources of risk
- Each factor portfolio has $\beta=1$ for one of the factors and 0 for all other factors (important)
Where Should We Look for Factors?

• Need important systematic risk factors
  – Chen, Roll, and Ross used industrial production, expected inflation, unanticipated inflation, excess return on corporate bonds, and excess return on government bonds.
  – Fama and French used firm characteristics that proxy for systematic risk factors.
Fama-French Three-Factor Model

- **SMB** = Small Minus Big (return of small in excess of big firms, based on firm size)
- **HML** = High Minus Low (return of firms with high Book-to-Market ratio, over those with low BtM)
- Are these firm characteristics correlated with actual (but currently unknown) systematic risk factors?

\[
R_{i,t} = \alpha_i + \beta_{i,M} R_{Mt} + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + e_{i,t}
\]
The Multifactor CAPM and the APT

• A multi-index CAPM will inherit its risk factors from sources of risk that a broad group of investors deem important enough to hedge

• The APT is largely silent on where to look for priced sources of risk